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Deep inelastic scattering with tagged initial state radiation: Complete $\mathcal{O}(\alpha)$ leptonic QED corrections

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Abstract. In this paper we extend the calculation of the QED corrections to deep inelastic lepton-proton scattering with a tagged photon, taking into account the full corrections on the lepton side. Comparing to previous results that were obtained by considering only large logarithmic terms at leading and next-to-leading accuray, we find that the difference is in general quite small, however, it may be significant in the region of large y and small x.

1 Introduction

One of the major aims of the experiments at the HERA ep collider is the measurement of the structure functions of the proton, $F_2(x, Q^2)$ and $F_L(x, Q^2)$, over a broad range of the kinematic variables. Especially the domain of small Bjorken $x < 10^{-4}$ and momentum transfer Q^2 of the order of a few GeV^2 and below is of particular interest, as it provides a challenge for attempts towards a complete, quantitative understanding of the dynamics of quarks and gluons inside the nucleon.

Of these structure functions, the longitudinal one, F_L, is much more difficult to access. There exist several ways to separately extract F_2 and F_L from the experimental data. The most obvious, direct method requires to run the collider at different (i.e., lower) center-of-mass energies, which may not be desirable from the point of view of other parts of the physics program.

Indirect methods usually require substantial input from theory and depend more or less on modeling the hadronic final state, like extrapolations or QCD fits (see e.g., [1–4]), or the measurement of the azimuthal angle distribution of final state hadrons, as suggested by Gehrmann [5]. However, these methods can be used at fixed collision energy.

Another direct method was suggested by Krasny et al. [6] and utilizes radiative events with an exclusive hard photon registered in the forward photon detector (PD). Such a device is actually part of the luminosity monitoring system of the H1 and ZEUS experiments, and will continue to exist after the HERA luminosity upgrade. The idea of this method is that emission of photons in a direction close to the incoming electron corresponds to a reduction in the effective beam energy. The effective electron energy for each radiative event is determined from the energy of the hard photon observed (tagged) in the PD.

Besides measuring F_L, radiative events extend the accessible kinematic range to lower values of Q^2. The potential of this method is supported by preliminary results from the H1 collaboration of an analysis at low Q^2 for F_2 [7–10] (for earlier analyses that did not take into account QED radiative corrections see [11,12]). The feasibility of the corresponding determination of F_L was studied in [13]. However, with currently analyzed data sets it is not yet possible to compete with F_L from extrapolations or QCD fits [8].

A precise analysis of experimental data requires the inclusion of radiative corrections. The most important ones are the QED corrections on the lepton side that have been discussed at the leading logarithmic level [14,15] and taking into account next-to-leading logarithms [16–18]. As the difference between leading and next-to-leading logarithmic terms could be quite significant, reaching of the order of 5 percent in some regions, it appears difficult to estimate the remaining uncertainty due to non-logarithmic terms. It is the purpose of the present work to close this particular gap.

We may focus on the QED corrections on the lepton side since they form a gauge invariant subset of the full corrections to the process under consideration. Furthermore, one expects that the QED corrections on the hadronic side are significantly smaller, as the typical hadronic mass scale of the order of the proton mass is much larger than the electron mass, so there is no comparably large logarithm. In fact, emission of photons off quarks can essentially be absorbed into suitably defined parton distribution functions, and the net effect turns out to be numerically negligible [19].

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The outline of the present paper is as follows. After introducing our notation in Sect. 2, we devote Sect. 3 to a discussion of the individual contributions to the leptonic corrections with forward photon tagging, and apply these corrections in Sect. 4 to the case of radiative deep inelastic scattering. In Sect. 5 we present some numerical results and our conclusions. The appendices collect several formulae that are useful for a numerical implementation of our calculation.

2 Born cross section

A a starting point, let us introduce our notation in the context of the lowest order contribution to the radiative, semi-inclusive deep inelastic scattering process

$$e(p) + p(P) \to e(p') + \gamma(k) + X(P') , \tag{1}$$

where the photon γ is assumed to be measured in the photon detector.

We choose invariant kinematic variables that use the measured, scattered electron and take into account the energy loss due to photon emission [6],

$$\begin{aligned} \hat{Q}^2 &= -(p-p'-k)^2 , \\ \hat{x} &= \frac{\hat{Q}^2}{2P\cdot(p-p'-k)} , \\ \hat{y} &= \frac{P\cdot(p-p'-k)}{P\cdot(p-k)} . \end{aligned} \tag{2}$$

For HERA conditions, the polar angle ϑ_γ of the tagged photon (as measured with respect to the incident electron beam) will be very small, $\vartheta_\gamma \le \vartheta_0$, with ϑ_0 being about 0.45 mrad in the case of the PD of H1. We shall also assume below that the scattering angle of the electron, θ, is always much larger than ϑ_0. Therefore, the energy fraction of the electron after initial state radiation reads

$$z = \frac{2P\cdot(p-k)}{S} = \frac{E_e - E_\gamma}{E_e} = \frac{\hat{Q}^2}{\hat{x}\hat{y}S} , \tag{3}$$

where E_e is the electron beam energy, E_γ represents the energy deposited in the forward PD, and $S = 2P\cdot p$.

The relation between the shifted variables (2) and the standard Bjorken variables of deep inelastic scattering reads:

$$Q^2 = \frac{\hat{Q}^2}{z} , \qquad x = \frac{\hat{x}\hat{y}}{1-z(1-\hat{y})} , \qquad y = 1 - z(1-\hat{y}) . \tag{4}$$

The Born cross section, integrated over the solid angle of the photon detector ($0 \le \vartheta_\gamma \le \vartheta_0$, $m/E_e \ll \vartheta_0 \ll 1$) takes a factorized form,

$$\frac{1}{\hat{y}} \frac{\mathrm{d}^3\sigma_{\mathrm{Born}}}{\mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}z} = \frac{\alpha}{2\pi} P(z, L_0)\, \tilde{\Sigma}(\hat{x},\hat{y},\hat{Q}^2) , \tag{5}$$

with

$$\tilde{\Sigma}(\hat{x},\hat{y},\hat{Q}^2) = \frac{2\pi\alpha^2(-\hat{Q}^2)}{\hat{Q}^2\hat{x}\hat{y}^2} \left[2(1-\hat{y}) - 2\hat{x}^2\hat{y}^2\frac{M^2}{\hat{Q}^2} + \left(1 + 4\hat{x}^2\frac{M^2}{\hat{Q}^2}\right)\frac{\hat{y}^2}{1+R} \right] F_2(\hat{x},\hat{Q}^2) , \tag{6}$$

and

$$\begin{aligned} P(z, L_0) &= \frac{1+z^2}{1-z} L_0 - \frac{2z}{1-z} , \qquad L_0 = \ln\left(\frac{E_e^2\vartheta_0^2}{m^2}\right) , \\ \alpha(-\hat{Q}^2) &= \frac{\alpha}{1 - \Pi(-\hat{Q}^2)} , \\ R = R(\hat{x},\hat{Q}^2) &= \left(1 + 4\hat{x}^2\frac{M^2}{\hat{Q}^2}\right) \frac{F_2(\hat{x},\hat{Q}^2)}{2\hat{x}F_1(\hat{x},\hat{Q}^2)} - 1 . \end{aligned} \tag{7}$$

The obvious advantage of the use of shifted variables (2) is their direct appearance in the argument of the structure functions in the Born cross section.

3 Leptonic corrections

In the expression for the lowest order cross section, (5), we encountered the logarithm $L_0 =: \ln\zeta_0$ of the quantity

$$\zeta_0 = \frac{E_e^2\vartheta_0^2}{m^2} . \tag{8}$$

Although $\vartheta_0 \ll 1$, we have $\zeta_0 \gg 1$ and $L_0 \approx 6.5 \gg 1$ for the conditions of the HERA PD's. We shall therefore consistently neglect contributions that are of order $\mathcal{O}(\vartheta_0)$ or $\mathcal{O}(\zeta_0^{-1})$.

As explained in the introduction, the subset of leptonic QED corrections is gauge invariant, and it also factorizes, thus allowing a discussion isolated from the hadronic part. Besides keeping things more transparent, this also facilitates reusing the results in other calculations.

In the present section, we shall therefore consider the Compton subprocess

$$e(p_1) + \gamma^*(-q) \to e(p_2) + \gamma(k) , \tag{9}$$

with the emission angle of the photon being integrated over the PD, while taking into account the corrections from virtual and real QED corrections. While performing this integration, we require that the remaining part of the amplitude for the full process (i.e., $\gamma^* + p \to X$) depends only weakly on the small transverse momentum of the forward photon.

3.1 Compton tensor

Let M_μ be the matrix element of the Compton scattering process (9), with the index μ describing the polarization state of the virtual photon. Adopting the notation of [20], we define the Compton tensor

$$K_{\mu\nu} = \frac{1}{(2e^2)^2} \sum_{\mathrm{spins}} M_\mu^{e\gamma^*\to e'\gamma} (M_\nu^{e\gamma^*\to e'\gamma})^* . \tag{10}$$

Using current conservation, this tensor is conveniently decomposed as follows:

$$K_{\mu\nu} = \frac{1}{2}\left(P_{\mu\nu} + P^*_{\nu\mu}\right) , \tag{11}$$

$$P_{\mu\nu} = \tilde{g}_{\mu\nu}\left(B_g + \frac{\alpha}{2\pi}T_g\right) + \sum_{i,j=1,2} \tilde{p}_{i\mu}\tilde{p}_{j\nu}\left(B_{ij} + \frac{\alpha}{2\pi}T_{ij}\right) ,$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} , \quad \tilde{p}_{i\mu} = p_{i\mu} - q_\mu \frac{p_i\cdot q}{q^2} , \quad i = 1, 2 .$$

The expressions for the quantities B_{ij} corresponding to the Born approximation are:

$$\begin{aligned}
B_g &= \frac{1}{\hat{s}\hat{t}}\left[(\hat{s}+\hat{u})^2 + (\hat{t}+\hat{u})^2\right] - 2m^2q^2\left(\frac{1}{\hat{s}^2} + \frac{1}{\hat{t}^2}\right) ,\\
B_{11} &= \frac{4q^2}{\hat{s}\hat{t}} - \frac{8m^2}{\hat{s}^2} ,\\
B_{22} &= \frac{4q^2}{\hat{s}\hat{t}} - \frac{8m^2}{\hat{t}^2} , \quad B_{12} = B_{21} = 0 ,\\
&\hat{s} = 2p_2\cdot k , \quad \hat{t} = -2p_1\cdot k ,\\
&\hat{u} = (p_1-p_2)^2 , \quad \hat{s}+\hat{t}+\hat{u} = q^2 .
\end{aligned} \tag{12}$$

Note that for almost collinear emission, $k \simeq (1-z)p_1$, we may neglect the transverse momentum of the emitted photon in the tensor decomposition (11) and use momentum conservation to set $\tilde{p}_2 = z\tilde{p}_1$.

The kinematic variables of the Compton subprocess are related to those of the radiative DIS process via:

$$\hat{u} = -\frac{\hat{Q}^2}{z} , \quad q^2 = (p_1-k-p_2)^2 \simeq -\hat{Q}^2 , \quad \hat{s} \simeq \frac{1-z}{z}\hat{Q}^2 .$$

The quantities T_g, T_{ij} in (11) denote the radiative corrections to the Compton tensor.

3.2 Virtual and soft corrections

The virtual corrections to the Compton tensor, as calculated in [20], are conveniently decomposed into a piece containing the universal infrared singular contributions, which are proportional to the Born contributions, and an infrared finite remainder:

$$T_g = \rho B_g + T'_g , \quad T_{ij} = \rho B_{ij} + T'_{ij} , \quad i,j = 1,2 , \tag{13}$$

where

$$\begin{aligned}
\rho &= 4\ln\frac{\lambda}{m}(L_Q-1) - L_Q^2 + 3L_Q + 3\ln z + \frac{\pi^2}{3} - \frac{9}{2} ,\\
L_Q &= \ln\frac{Q^2}{m^2} .
\end{aligned} \tag{14}$$

The parameter λ in the above expression is a fictitious photon mass regulating the IR divergency.

Performing the integration over photon angles, we obtain

$$\begin{aligned}
\frac{E_e^2}{\pi}\int \mathrm{d}\Omega_k\, B_{\mu\nu} &= \left(-Q_l^2\tilde{g}_{\mu\nu} + 4z\tilde{p}_{1\mu}\tilde{p}_{1\nu}\right)\\
&\quad\times \frac{1}{1-z}\left[\left(1+\frac{\alpha}{2\pi}\rho\right)P(z,L_0) - \frac{\alpha}{2\pi}T\right]\\
&\quad+ \mathcal{O}\left(\vartheta_0^2, \zeta_0^{-1}\right) ,
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
T &= (A\ln z + B)P(z,L_0) + CL_0 + D ,\\
A &= 2L_Q - L_0 - 2\ln(1-z) ,\\
B &= \ln^2 z - 2\,\mathrm{Li}_2(1-z) - \frac{1}{2} ,\\
C &= -\frac{2z}{1-z}\ln z - z ,\\
D &= -\frac{1-6z+4z^2}{1-z}\left(\mathrm{Li}_2(1-z) + \ln z\ln(1-z)\right)\\
&\quad - 2z\ln^2(1-z) + \frac{8z}{1-z}\ln z - \frac{4\pi^2}{3}z + 1 ,\\
\mathrm{Li}_2(x) &= -\int_0^x \frac{\mathrm{d}y}{y}\ln(1-y) .
\end{aligned}$$

The single and double logarithmic terms in L_0 and L_Q of the above expression agree with [17]. Details of the calculation will be given elsewhere [21].

The dependence of the virtual corrections on the unphysical parameter λ is canceled by the contribution from emission of an additional soft photon, as usual. Requiring that the energy fraction of the second (soft) photon in units of the energy of the incoming electron does not exceed ϵ, with $\epsilon \ll 1$, and adding the contribution from soft photon emission to the virtual correction then amounts to the replacement of the quantity ρ in (15) by $\tilde{\rho}$, see [20]:

$$\begin{aligned}
\tilde{\rho} &= 2(L_Q-1)\ln\frac{\epsilon^2}{Y} + 3L_Q + 3\ln z - \ln^2 Y - \frac{\pi^2}{3} - \frac{9}{2}\\
&\quad+2\,\mathrm{Li}_2\left(\frac{1+c}{2}\right) ,
\end{aligned} \tag{16}$$

with

$$Y = \frac{E'_e}{E_e} \quad \text{and} \quad c = \cos\theta = \cos\measuredangle(\vec{p},\vec{p}\,') \tag{17}$$

being the relative energy of the scattered electron and the cosine of the scattering angle in the lab system, respectively.

3.3 Double hard bremsstrahlung

For the case of double photon emission, we define the 'double Compton tensor' analogously to (10) as

$$K^{\gamma\gamma}_{\mu\nu} = \frac{1}{2e^6}\sum_{\text{spins}} M_\mu^{e\gamma^*\to e'\gamma\gamma}\left(M_\nu^{e\gamma^*\to e'\gamma\gamma}\right)^* , \tag{18}$$

where now $M_\mu^{e\gamma^*\to e'\gamma\gamma}$ is the matrix element of the double Compton process process

$$e(p_1) + \gamma^*(-q) \to e(p_2) + \gamma(k_1) + \gamma(k_2) , \tag{19}$$

with the index μ describing the polarization of the virtual photon.

For the kinematic invariants of this subprocess we shall use the notation:

$$\begin{aligned}
z_i &= 2p_1 \cdot k_i \ , \quad z_i' = 2p_2 \cdot k_i \ , \\
\sigma &= 2k_1 \cdot k_2 = (k_1 + k_2)^2 \ , \\
\Delta &= -[(p_1 - k_1 - k_2)^2 - m^2] = z_1 + z_2 - \sigma \ , \\
\Delta' &= [(p_2 + k_1 + k_2)^2 - m^2] = z_1' + z_2' + \sigma \ , \\
Q_l^2 &= -(p_1 - p_2)^2 = 2(p_1 \cdot p_2 - m^2) \ , \\
Q_h^2 &= -q^2 = Q_l^2 + z_1 + z_2 - z_1' - z_2' - \sigma \ .
\end{aligned}$$

3.3.1 Double collinear emission

When both photons are emitted almost collinearly to the incoming electron, ($k_1 \simeq x_1 p_1$, $k_2 \simeq x_2 p_1$), we may neglect the transverse momenta of the photons with respect to the incoming electron, and the double Compton tensor takes a simple form:

$$\begin{aligned}
K_{\mu\nu}^{2-\mathrm{coll}} &= 4\left[-\tilde{g}_{\mu\nu} Q_l^2 + 4z\,(\tilde{p}_{1\mu}\tilde{p}_{1\nu})\right] \\
&\times \left[\frac{1+z^2}{x_1 x_2}\frac{1}{z_1 z_2} - \frac{z}{\Delta^2}\left(\frac{z_1}{z_2} + \frac{z_2}{z_1}\right)\right. \\
&+ \frac{1}{x_1 x_2}\left(\frac{r_1^3 + z r_2}{z_1 \Delta} + \frac{r_2^3 + z r_1}{z_2 \Delta}\right) \\
&- 2\frac{m^2}{\Delta}\left(\frac{r_1^2 + z^2}{x_2 z_1^2} + \frac{r_2^2 + z^2}{x_1 z_2^2} + \frac{(1-z)(r_1 r_2 + z)}{x_1 x_2 z_1 z_2}\right) \\
&\left. - 4z\,\frac{m^2}{\Delta^2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right) + 4z\,\frac{m^4}{\Delta^2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)^2\right] . \qquad (20)
\end{aligned}$$

Here $r_1 = 1 - x_1$, $r_2 = 1 - x_2$, and $z = 1 - x_1 - x_2$. This expression is consistent with Merenkov [22], where leading and next-to-leading logarithms were calculated.

Performing the integration over the photon angles and over the relative photon energies, taking into account the symmetry factor $1/2!$ for the emitted photons, one obtains:

$$\begin{aligned}
&\frac{e^4}{2!}\int \widetilde{\mathrm{d}k}_1\,\widetilde{\mathrm{d}k}_2\;\Theta(x_1 - \epsilon)\,\Theta(x_2 - \epsilon)\;\delta(x_1 + x_2 - (1-z)) \\
&\quad \times K_{\mu\nu}^{2-\mathrm{coll}} = \left[-\tilde{g}_{\mu\nu} Q_l^2 + 4z\,(\tilde{p}_{1\mu}\tilde{p}_{1\nu})\right] \\
&\quad \times \frac{\alpha^2}{8\pi^2}\left[P_{\mathrm{log}}^{(2)} + P_{\mathrm{nonlog}}^{(2),\mathrm{IR-div.}} + P_{\mathrm{nonlog}}^{(2),\mathrm{IR-fin.}}\right] . \qquad (21)
\end{aligned}$$

The previously known leading terms containing double and single logarithms L_0 are contained in $P_{\mathrm{log}}^{(2)}$. The non-leading terms are split into an infrared divergent piece that depends on $\ln\epsilon$, and an infrared finite piece. An outline of the calculation and expressions are given in appendix A.

It is worth to mention that the terms depending on the soft-photon cutoff parameter ϵ in (21) do factor nicely, as expected from the usual soft-photon factorization:

$$\left[-\tilde{g}_{\mu\nu} Q_l^2 + 4z\,(\tilde{p}_{1\mu}\tilde{p}_{1\nu})\right] \times \left(\frac{\alpha}{2\pi}\right)^2 P(z, L_0)\cdot 2(L_0 - 1)\ln\frac{1}{\epsilon} \ . \qquad (22)$$

3.3.2 Final state collinear radiation

Consider now the kinematic region where one photon, say, photon 1, is emitted almost collinearly to the incoming electron, i.e., $k_1 \simeq x_1 p$, and the other close to the outgoing electron, so that $k_2 \simeq \xi(p_2 + k_2)$. The double Compton tensor then simplifies to

$$\begin{aligned}
K_{\mu\nu}^{\mathrm{FSR}} &\simeq \left[-\tilde{g}_{\mu\nu}\frac{Q_l^2}{1-\xi} + 4(1-x_1)\cdot(\tilde{p}_{1\mu}\tilde{p}_{1\nu})\right] \\
&\times 2\left[\frac{1+(1-x_1)^2}{x_1}\frac{1}{z_1} - 2(1-x_1)\frac{m^2}{z_1^2}\right] \\
&\times 2\left[\frac{1+(1-\xi)^2}{\xi}\frac{1}{z_2'} - 2\frac{m^2}{{z_2'}^2}\right] ,
\end{aligned}$$

exhibiting the expected complete factorization of collinear initial and final state radiation, respectively.

Integrating this expression over the emission angles of photon 1, one obtains

$$\begin{aligned}
\frac{E^2}{\pi}\int \mathrm{d}\Omega_1\;K_{\mu\nu}^{\mathrm{FSR}} &= \left[-\tilde{g}_{\mu\nu}\frac{Q_l^2}{1-\xi} + 4(1-x_1)\cdot(\tilde{p}_{1\mu}\tilde{p}_{1\nu})\right] \\
&\times \frac{2}{x_1}\,P(1-x_1, L_0) \\
&\times 2\left[\frac{1+(1-\xi)^2}{\xi}\frac{1}{z_2'} - 2\frac{m^2}{{z_2'}^2}\right] .
\end{aligned}$$

Note that we have not yet integrated over the angles of the final state photon. The treatment of final state radiation depends on the experimental situation, i.e., whether the detector is able to resolve a photon collinear to the electron, or whether it just measures the sum of their energies.

3.3.3 Semi-collinear emission

The final case covers the kinematic range where one photon is emitted almost collinearly to the incoming electron, while the other photon is emitted at an angle $\vartheta_2 > \vartheta_0$, but not collinear to the final electron. We denote this kinematic domain as the semi-collinear one.

In order to be consistent with the above calculation of the double collinear emission, we shall perform the angular integration over the collinear photon and drop all contributions of the order $\mathcal{O}(\vartheta_0)$ and $\mathcal{O}(\zeta_0^{-1})$. We find indeed factorization of initial state radiation for large emission angles of the second photon, i.e., $\vartheta_2 \gg \vartheta_0$. However, in the vicinity of the forward cone that is defined by the solid angle of the PD, there are contributions from further terms with a complicated z_2-dependence that spoil a naive factorization. These additional terms fall off rapidly and essentially contribute only in the small region $\vartheta_0 < \vartheta_2 \lesssim 2\vartheta_0$.

Assuming that photons in this narrow region outside the PD will not be measured, we integrate these terms over angles and split their contribution schematically as follows:

$$
\begin{aligned}
&\frac{E^4}{\pi^2} \int\limits_{\vartheta_1<\vartheta_0} \mathrm{d}\Omega_1 \int\limits_{\vartheta_2>\vartheta_0} \mathrm{d}\Omega_2\, K_{\mu\nu}^{\mathrm{semi-coll}} \qquad (23)\\
&\simeq \frac{E^2}{\pi} \int\limits_{\vartheta_2>\vartheta_0} \mathrm{d}\Omega_2 \left\{ \left[-\tilde{g}^{\mu\nu} \frac{(r_1(Q_l^2+z_2))^2 + (r_1 Q_l^2 - z_2')^2}{r_1 z_2 z_2'} \right.\right.\\
&\qquad \left.\left. + 4 r_1^2 \tilde{p}_1^\mu \tilde{p}_1^\nu \frac{Q_h^2}{r_1 z_2 z_2'} + 4 \tilde{p}_2^\mu \tilde{p}_2^\nu \frac{Q_h^2}{r_1 z_2 z_2'} \right] \cdot \frac{4}{x_1 r_1}\, P(r_1, L_0) \right\}\\
&\qquad + \left[-\tilde{g}_{\mu\nu} Q_l^2 + 4(r_1 - x_2)\, (\tilde{p}_{1\mu} \tilde{p}_{1\nu}) \right] \cdot \frac{4}{x_1 x_2}\, H(x_1, x_2)\ ,
\end{aligned}
$$

with $r_1 = 1 - x_1$. The explicit expression for the function $H(x_1, x_2)$, which collects the mentioned non-factorizing, quasi-collinear terms, is given in appendix B. It is infrared-finite and does not contain any logarithm of a large scale.

The integrand in the first part on the r.h.s. of (23) can be rewritten as

$$
\left\{ \ldots \right\} \simeq \frac{1}{x_1}\, P(r_1, L_0) \cdot \frac{4}{r_1} B_{\mu\nu}^{\mathrm{Born}}(r_1 p_1, p_2)\ , \qquad (24)
$$

where for the sake of consistency one should drop terms of order m^2 on the r.h.s. For a discussion and further details see [21].

Since in our decomposition of phase space only photon 1 reaches the PD, we have to identify r_1 by z and x_1 by $1 - z$ in the above expressions. However, we still need to integrate over the phase space of the other photon that is emitted at large angles. This calculation depends on the complete scattering process and in general requires a numerical integration.

4 Radiative DIS

Let us now turn to the description of the radiative scattering process

$$
e(p) + p(P) \to e(p') + X(P') + \gamma(k_1)\ \ (+\gamma(k_2))\ .
$$

As argued in the introduction, the leading raditiative corrections to the process stem from emission of photons off the lepton line[1]. It is now straightforward to contract the radiatively corrected Compton tensor of the previous section with the hadron tensor

$$
\begin{aligned}
H_{\mu\nu}(P, q_h) &= 4\pi \Bigg(- \tilde{g}_{\mu\nu} F_1(x_h, Q_h^2)\\
&\qquad + \tilde{P}_\mu \tilde{P}_\nu \frac{1}{P \cdot q_h} F_2(x_h, Q_h^2) \Bigg)\\
&= 4\pi \Bigg(- \tilde{g}_{\mu\nu} F_1(x_h, Q_h^2)\\
&\qquad + \tilde{P}_\mu \tilde{P}_\nu \frac{2 x_h}{Q_h^2} F_2(x_h, Q_h^2) \Bigg)\ ,\\
\tilde{P}_\nu &= P_\nu - q_{h\nu} \frac{P \cdot q_h}{q_h^2}\ , \quad x_h = \frac{Q_h^2}{2 P \cdot q_h}\ ,\\
Q_h^2 &= -q_h^2\ .
\end{aligned}
$$

Here q_h denotes the four-momentum transfer to the hadronic system. Taking into account the emission of two photons, $q_h = p_1 - p_2 - k_1 - k_2$.

Applying the results from the previous section, we find for the contribution from virtual and soft corrections to the cross section:

$$
\frac{1}{\hat{y}} \frac{\mathrm{d}^3 \sigma_{\mathrm{V+S}}}{\mathrm{d}\hat{x}\, \mathrm{d}\hat{y}\, \mathrm{d}z} = \frac{\alpha^2}{4\pi^2} \left[P(z, L_0) \tilde{\rho} - T \right] \tilde{\Sigma}(\hat{x}, \hat{y}, \hat{Q}^2)\ , \qquad (25)
$$

where $\tilde{\rho}$ is taken from (16) with

$$
\begin{aligned}
Y &= \frac{E_e'}{E_e} = z(1 - \hat{y}) + \hat{x}\hat{y} \frac{E_p}{E_e}\ ,\\
c &= \cos\theta = \frac{z(1-\hat{y}) E_e - \hat{x}\hat{y} E_p}{z(1-\hat{y}) E_e + \hat{x}\hat{y} E_p}\ . \qquad (26)
\end{aligned}
$$

In the calculation of the contributions from the emission of two hard photons, we decompose the phase space into three regions discussed in the previous section (see also [17]): *i)* both hard photons hit the forward photon detector, i.e., both are emitted within a narrow cone around the electron beam ($\vartheta_{1,2} \le \vartheta_0$, $\vartheta_0 \ll 1$); *ii)* one photon is tagged in the PD, while the other is collinear to the outgoing electron ($\vartheta_2' \equiv \measuredangle(\vec{k}_2, \vec{p}\,') \le \vartheta_0'$); and finally *iii)* the second photon is emitted at large angles (i.e., outside the defined narrow cones) with respect to both incoming and outgoing electron momenta. For the sake of simplicity, we assume that $m/E_e \ll \vartheta_0' \ll 1$.

The contribution from the kinematic region *i)*, with both hard photons being tagged in the PD, but only the sum of their energies measured, reads:

$$
\frac{1}{\hat{y}} \frac{\mathrm{d}^3 \sigma_i^{\gamma\gamma}}{\mathrm{d}\hat{x}\, \mathrm{d}\hat{y}\, \mathrm{d}z} = \frac{\alpha^2}{8\pi^2} \left[P_{\mathrm{log}}^{(2)} + P_{\mathrm{nonlog}}^{(2),\mathrm{IR-div.}} + P_{\mathrm{nonlog}}^{(2),\mathrm{IR-fin.}} \right] \tilde{\Sigma}\ , \qquad (27)
$$

see (21).

In region *ii)* we need to distinguish between the cases of whether a photon emitted close to the outgoing electron can be detected separately (exclusively), or whether its energy and momentum are measured together with the electron (inclusively), as this affects the reconstructed kinematic variables.

For the exclusive event selection, when only the scattered electron is detected, we obtain

$$
\begin{aligned}
\frac{1}{\hat{y}} \frac{\mathrm{d}^3 \sigma_{ii,\mathrm{excl}}^{\gamma\gamma}}{\mathrm{d}\hat{x}\, \mathrm{d}\hat{y}\, \mathrm{d}z} &= \frac{\alpha^2}{4\pi^2} P(z, L_0) \int\limits_{\zeta_{\min}}^{\zeta_{\max}} \frac{\mathrm{d}\zeta}{\zeta^2}\\
&\quad \times \left[\frac{1+\zeta^2}{1-\zeta} \left(\widetilde{L} - 1 \right) + 1 - \zeta \right] \tilde{\Sigma}_f\ , \qquad (28)
\end{aligned}
$$

[1] Another gauge invariant set of large corrections is due to the running of the QED coupling α. Note that we have already included it in the expression for the Born cross section, so we won't discuss it further

where $\tilde{L} = \ln(E_e'^2 \vartheta_0'^2/m^2) = \ln\left(E_e^2 \vartheta_0'^2/m^2\right) + 2\ln Y$, and $\tilde{\Sigma}_f = \tilde{\Sigma}(x_f, y_f, Q_f^2)$ is an implicit function of ζ via the relation between the "internal" kinematic variables x_f, y_f, Q_f^2 and the "external" ones $\hat{x}, \hat{y}, \hat{Q}^2$ (see [16–18] for more details). The integration limits explicitly depend on the method for the determination of kinematic variables.

In the case of a calorimetric event selection, where only the sum of the energies of the outgoing electron and collinear photon is measured and taken into account in the determination of the kinematic variables, the corresponding contribution reads

$$\frac{1}{\hat{y}} \frac{\mathrm{d}^3\sigma_{ii,\mathrm{cal}}^{\gamma\gamma}}{\mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}z} = \frac{\alpha^2}{4\pi^2} P(z, L_0) \int\limits_0^{\zeta_{\max}} \mathrm{d}\zeta \times \left[\frac{1+\zeta^2}{1-\zeta}\left(\tilde{L} - 1 + 2\ln\zeta\right) + 1 - \zeta\right] \tilde{\Sigma}\,, \tag{29}$$

see also [16–18].

For the contribution from the semi-collinear region *iii)* we apply the results from section 3.3.3 to obtain:

$$\frac{1}{\hat{y}} \frac{\mathrm{d}^3\sigma_{iii}^{\gamma\gamma}}{\mathrm{d}\hat{x}\,\mathrm{d}\hat{y}\,\mathrm{d}z} = \frac{\alpha^2}{\pi^2} P(z, L_0) \int \frac{\mathrm{d}^3 k_2}{|\vec{k}_2|} \frac{\alpha^2(Q_h^2)}{Q_h^4} I^\gamma(zp, p', k_2) + \frac{\alpha^2}{4\pi^2} \int\limits_\epsilon^{x_2^t} \mathrm{d}x_2 \, \frac{z}{z-x_2} H(1-z, x_2) \times \tilde{\Sigma}(x_t, y_t, Q_t^2)\,, \tag{30}$$

where the 'radiation kernel' I^γ and the boundaries of integration for the logarithmic part are given in [17]. For the second, quasi-collinear contribution, we have used the abbreviations

$$x_t = \frac{(z-x_2)\hat{x}\hat{y}}{z\hat{y} - x_2}\,, \quad y_t = \frac{z\hat{y} - x_2}{z - x_2}\,, \quad Q_t^2 = \hat{Q}^2 \frac{z-x_2}{z}\,, \tag{31}$$

and the upper limit of integration is

$$x_2^t = z\hat{y}\,. \tag{32}$$

The total contribution from QED radiative corrections is finally obtained by adding up (25), (27), (30), and, depending on the chosen event selection, (28) or (29). One easily verifies that the unphysical IR regularization parameter ϵ cancels in the sum.

5 Results and discussion

In this section we shall present numerical results obtained from the above expressions and compare to next-to-leading radiative corrections [17]. As input we used

$$E_e = 27.5 \text{ GeV}\,, \quad E_p = 820 \text{ GeV}\,, \quad \vartheta_0 = 0.5 \text{ mrad}\,. \tag{33}$$

We chose the ALLM97 parameterization [23] as structure function with $R = 0$, no cuts were applied to the phase space of the second photon, and we assumed a calorimetric event selection. Furthermore, we took a fixed representative angular resolution of $\vartheta_0' = 50\,\mathrm{mrad}$.

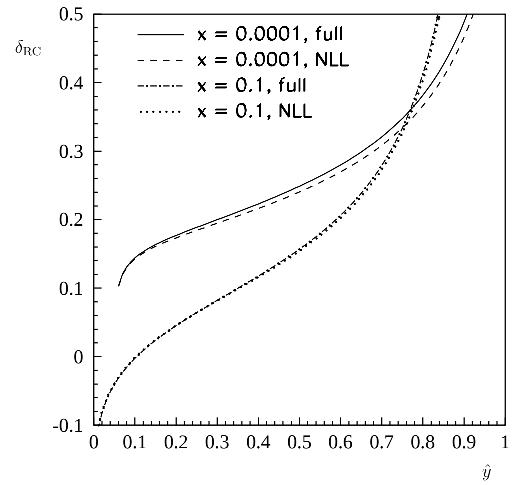

Fig. 1. Radiative corrections δ_{RC} (34) with full leptonic $\mathcal{O}(\alpha)$ vs. next-to-leading logarithmic accuracy at $\hat{x} = 10^{-4}$ and $\hat{x} = 0.1$ and a tagged photon energy of 20 GeV. No cuts have been applied to the phase space of the second (semi-collinear) photon

Figure 1 compares the radiative correction

$$\delta_{\mathrm{RC}} = \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3\sigma_{\mathrm{Born}}} - 1\,, \tag{34}$$

calculated with next-to-leading logarithmic and full $\mathcal{O}(\alpha)$ accuracy for the electron method at $\hat{x} = 10^{-4}$ and $\hat{x} = 0.1$ and for a tagged energy of $E_{\mathrm{PD}} = 20$ GeV. Similar to the well-known QED corrections to DIS (see e.g., [24] and references cited therein), the corrections are large and positive for large $\hat{y}$, while they may become large and negative for $\hat{y} \to 0$ at large $\hat{x}$.

Furthermore, one sees that the difference between the full result and the one at next-to-leading logarithmic accuracy [17] is very small; it is typically at the permille level except for large $\hat{y}$, where the phase space for the 'lost photon' gets large, see (32), and for small $\hat{x}$. In this case it easily reaches values of the order of a percent. This may be important in view of the increased statistics expected at the upgraded HERA collider.

Looking at the individual contributions beyond the next-to-leading logarithmic approximation, one finds a significant cancellation between terms from virtual+soft corrections and from double collinear emission. Their sum is typically of the order permille, and it depends only on z, c.f. (25) and (27). Therefore the total difference can be mainly attributed to the contribution of photons emitted into a small region outside but close to the PD.

To summarize, we have calculated the full leptonic QED corrections to DIS with tagged initial state radiation. We find that the corrections beyond next-to-leading logarithmic accuracy are typically very small, but can still be significant, i.e., of the order of a percent, in the interesting region of small x, where the use of radiative events appears most promising.

Appendix

A Integrals for double collinear emission

The calculation of the contribution from double collinear emission assumes that only the sum of the photon energies, $(1-z)E$, will be measured in the forward photon detector.

We split the integration of expression (21) over the restricted two-photon phase space in the following way:

$$\begin{aligned}
&\int \widetilde{\mathrm{d}k_1}\,\widetilde{\mathrm{d}k_2}\,\Theta(\vartheta_0-\vartheta_1)\,\Theta(\vartheta_0-\vartheta_2)\\
&\delta\left((1-z)-(x_1+x_2)\right)\left[\ldots\right]\\
&\quad=\frac{1}{(4\pi)^4}\int x_1\,\mathrm{d}x_1\int x_2\,\mathrm{d}x_2\\
&\quad\times\delta\left((1-z)-(x_1+x_2)\right)\overline{\left[\ldots\right]}\,. \qquad (35)
\end{aligned}$$

In the last line we adopted the notation of Arbuzov et al. [25],

$$\overline{\left[\ldots\right]} := \frac{E^4}{\pi^2}\int \mathrm{d}\Omega_1\,\mathrm{d}\Omega_2\,\Theta(\vartheta_0-\vartheta_1)\,\Theta(\vartheta_0-\vartheta_2)\left[\ldots\right], \qquad (36)$$

for the angular part of the integrals.

A.1 Integrals over photon angles

We shall now provide the relevant angular integrals needed for the contribution of two photons emitted almost collinearly to the incoming electron. The calculation is performed under the assumption that $E^2\vartheta_0^2/m^2 \gg 1$ and $\vartheta_0^2 \ll 1$. For details we refer the reader to [21].

Using the abbreviation

$$L_0 = \ln\frac{E^2\vartheta_0^2}{m^2}\,,$$

we obtain:

$$\begin{aligned}
\overline{\left[\frac{1}{z_1z_2}\right]} &= \frac{1}{x_1x_2}\,L_0^2\,,\\
\overline{\left[\frac{m^2}{z_1^2\Delta}\right]} &= \frac{1}{x_1^2x_2r_1}\left[L_0+\ln\frac{x_2r_1}{1-z}\right]\\
&\quad-\frac{z}{x_1x_2^2r_1}\ln\frac{r_1(1-z)}{x_1z}\,,\\
\overline{\left[\frac{1}{z_1\Delta}\right]} &= \frac{1}{x_1x_2r_1}\left[\frac{1}{2}L_0^2+L_0\ln\frac{x_2r_1^2}{x_1z}+\mathrm{Li}_2\left(-\frac{x_2}{x_1z}\right)\right.\\
&\quad\left.+\Xi\left(\cos\psi;\frac{x_1r_2}{x_2r_1}\right)\right],\\
\overline{\left[\frac{z_2}{z_1\Delta^2}\right]} &= \frac{1}{x_1x_2r_1^2}\left\{\frac{1}{2}L_0^2+L_0\left[\ln\frac{x_2r_1^2}{x_1z}-1+\frac{x_1x_2}{z}\right]\right.\\
&\quad+\mathrm{Li}_2\left(-\frac{x_2}{x_1z}\right)+\Xi\left(\cos\psi;\frac{x_1r_2}{x_2r_1}\right)\\
&\quad+\frac{r_1r_2-2z}{z}\ln(2x_2)+\frac{r_1(r_2-2z)}{z}\ln\frac{x_1}{1-z}\\
&\quad+\frac{r_1(3r_2-4z)}{2z}\ln z-\frac{r_1r_2+4x_1z}{2z}\ln r_1\\
&\quad-\frac{r_1r_2}{2z}\ln r_2\\
&\quad+\frac{4z-r_1r_2}{2z}\ln\left(\eta+x_2r_1+x_1r_2\cos\psi\right)\\
&\quad\left.-\frac{r_1r_2}{2z}\ln\left(\eta+x_1r_2+x_2r_1\cos\psi\right)\right\}, \qquad (37)
\end{aligned}$$

where $z = 1-x_1-x_2$, $r_{1,2} = 1-x_{1,2}$, and

$$\begin{aligned}
\cos\psi &= 1-\frac{2x_1x_2}{r_1r_2}\,,\\
\eta &= \sqrt{(x_1+x_2)(x_1+x_2-4x_1x_2)}\,,\\
\Xi(t;x) &= \frac{1}{2}\ln^2\left(\frac{\sqrt{1+2tx+x^2}+tx+1}{2}\right)\\
&\quad+\mathrm{Li}_2\left(\frac{(1+t)x}{\sqrt{1+2tx+x^2}+tx+1}\right)\\
&\quad+\mathrm{Li}_2\left(-\frac{(1-t)x}{\sqrt{1+2tx+x^2}+tx+1}\right). \qquad (38)
\end{aligned}$$

The integrals that contribute only non-enhanced terms (i.e., no L_0's) are:

$$\begin{aligned}
\overline{\left[\frac{m^2}{z_1z_2\Delta}\right]} &= \frac{1}{x_1^2x_2^2}\left[(1-z)\ln(1-z)+z\ln z-r_1\ln r_1\right.\\
&\quad\left.-r_2\ln r_2-x_1\ln x_1-x_2\ln x_2\right],\\
\overline{\left[\frac{m^2}{z_1\Delta^2}\right]} &= \frac{1}{x_1x_2^2}\ln\frac{r_1(1-z)}{x_1z}\,,\\
\overline{\left[\frac{m^4}{z_1^2\Delta^2}\right]} &= \frac{1}{x_1^2x_2^2}\left[1-\frac{x_1y}{x_2}\ln\frac{r_1(1-y)}{x_1y}\right],\\
\overline{\left[\frac{m^4}{z_1z_2\Delta^2}\right]} &= \frac{1}{6x_1^3x_2^3}\left[x_1^2(3-2x_1)\ln\frac{(1-z)r_1}{x_1z}\right.\\
&\quad+x_2^2(3-2x_2)\ln\frac{(1-z)r_2}{x_2z}\\
&\quad\left.+\ln\frac{z}{r_1r_2}-2x_1x_2\right]. \qquad (39)
\end{aligned}$$

The remaining integrals can be obtained from those given above by exchanging photons 1 and 2, i.e., $x_1 \leftrightarrow x_2$ and $r_1 \leftrightarrow r_2$.

It should be noted that the coefficients of the double and single logarithmic terms (L_0^2 and L_0) agree with [22, 25].

A.2 Integrals over relative photon energy

Having performed the angular integration, we still need to integrate over the relativ photon energy, see (35). As this integral will be infrared-divergent, we introduce a soft-photon cutoff ϵ on the minimum energy fraction of each photon, which is identical to the one used in the soft-photon contribution. We thus define:

$$\left\langle\left[\cdots\right]\right\rangle := \int_\epsilon^1 x_1\,\mathrm{d}x_1 \int_\epsilon^1 x_2\,\mathrm{d}x_2 \times\delta\left((1-z)-(x_1+x_2)\right)\left[\cdots\right] \tag{40}$$

With the substitution $x_1 \to (1-z)\cdot u$ and the abbreviation $\tilde\epsilon = \epsilon/(1-z)$, we have, after elimination of the trivial δ-function:

$$\left\langle\left[\cdots\right]\right\rangle = (1-z)\int_{\tilde\epsilon}^{1-\tilde\epsilon}\mathrm{d}u\; x_1\,x_2 \times\left[\cdots\right]_{x_1=(1-z)u,x_2=(1-z)(1-u),\ldots}$$

For the logarithmically (L_0) enhanced leading terms, we find the familar result:

$$\begin{aligned}P^{(2)}_{\log} &= \left[-4\frac{1+z^2}{1-z}\ln\tilde\epsilon + (1+z)\ln z - 2(1-z)\right]L_0^2 \\ &\quad+\left[6(1-z)+\frac{3+z^2}{1-z}\ln^2 z + 4\frac{(1+z)^2}{1-z}\ln\tilde\epsilon\right]L_0 \\ &= \left[P^{(2)}_{\Theta}(z) + 2\,\frac{1+z^2}{1-z}\left(\ln z - \frac{3}{2} - 2\ln\epsilon\right)\right]L_0^2 \\ &\quad+\left[6(1-z)+\frac{3+z^2}{1-z}\ln^2 z \right. \\ &\quad\left.+4\frac{(1+z)^2}{1-z}\ln\frac{\epsilon}{1-z}\right]L_0\,,\end{aligned}$$

with the leading-log radiator

$$P^{(2)}_{\Theta}(z) = 2\left[\frac{1+z^2}{1-z}\left(2\ln(1-z)-\ln z+\frac{3}{2}\right) +\frac{1}{2}(1+z)\ln z - 1 + z\right].$$

The analytical calculation of the remaining, non-enhanced terms is quite tedious, leading to lengthy expressions involving many dilogarithms and trilogarithms (see e.g., [26]).

As these terms also contain IR-divergent contributions, we shall pursue here the following approach. We analytically extract those terms in the integrand that either contribute to the infrared-divergence as $\epsilon \to 0$ or survive in the limit $z \to 1$, before performing the integral over the remaining expression numerically. Besides, this separation improves the stability of the numerical integration.

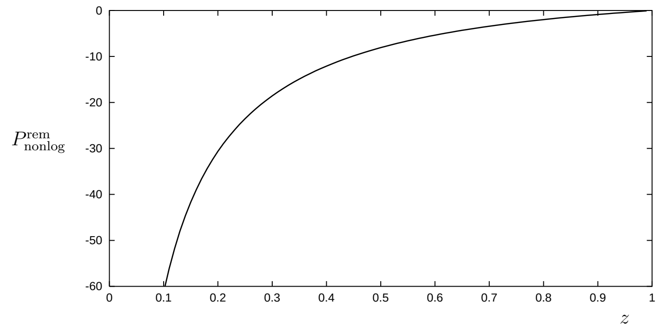

Fig. 2. The infrared-finite, 'normalized' part of the double-collinear contribution, $P^{\mathrm{rem}}_{\mathrm{nonlog}}(z)$

For the IR-divergent pieces of the non-enhanced terms we find

$$P^{\mathrm{IR-div}}_{\mathrm{nonlog}} = \left\langle\frac{4z}{(x_1x_2)^2}\right\rangle = \frac{4z}{1-z}\int_{\tilde\epsilon}^{1-\tilde\epsilon}\mathrm{d}u\,\frac{1}{u(1-u)} \simeq \frac{8z}{1-z}\ln\tilde\epsilon\,. \tag{41}$$

We then decompose the infrared-finite pieces as follows:

$$P^{(2),\mathrm{IR-fin.}}_{\mathrm{nonlog}}(z) = P^{z\to1}_{\mathrm{nonlog}} + P^{\mathrm{rem}}_{\mathrm{nonlog}}(z)\,, \tag{42}$$

so that $P^{\mathrm{rem}}_{\mathrm{nonlog}}(z=1)=0$. The first term on the r.h.s., obtained in the limit $z\to1$ reads:

$$\begin{aligned}P^{z\to1}_{\mathrm{nonlog}} &= \left\langle\frac{8}{3x_1x_2}\left[\frac{x_1+3x_2}{x_2^2}\ln x_1 + \frac{3x_1+x_2}{x_1^2}\ln x_2 \right.\right. \\ &\quad\left.\left.-\frac{(1-z)^3}{x_1^2x_2^2}\ln(1-z)+\frac{1-z}{x_1x_2}\right]\right\rangle \\ &= \int_{\tilde\epsilon}^{1-\tilde\epsilon}\mathrm{d}u\,\frac{8}{3}\left[\frac{1}{u(1-u)}+\frac{3-2u}{(1-u)^2}\ln u \right. \\ &\quad\left.+\frac{1+2u}{u^2}\ln(1-u)\right] \\ &= -\frac{16}{9}\left(3+\pi^2\right)\,,\end{aligned} \tag{43}$$

where we neglected terms of order ϵ.

Finally, we plot in Fig. 2 the 'remainder' $P^{\mathrm{rem}}_{\mathrm{nonlog}}(z)$, which we evaluated numerically.

B Non-factorizing quasi-collinear piece

In the discussion of the contributions to the semi-collinear piece, a non-factorizing part was split off in (23). The function H appearing there is given by

$$H(x_1,x_2) = \frac{r_1^3+\chi r_2}{x_1x_2r_1}H_1 + \frac{r_2^3+\chi r_1}{x_1x_2r_2}H_2 -\chi\left(\frac{H_3}{r_1^2}+\frac{H_4}{r_2^2}\right)\,, \tag{44}$$

with

$$H_1 = -\Xi\left(\cos\psi, \frac{x_1 r_2}{x_2 r_1}\right) ,$$

$$H_2 = \frac{1}{2}\ln^2\frac{x_1 r_2^2}{x_2\chi} + \frac{\pi^2}{6} + \mathrm{Li}_2\left(-\frac{x_1 x_2}{\chi}\right) - \Xi\left(\cos\psi, \frac{x_2 r_1}{x_1 r_2}\right) ,$$

$$H_3 = H_1 + \frac{2\cos\psi}{1+\cos\psi}\ln(2x_2 r_1) - \frac{1}{1+\cos\psi}\ln\frac{1+\cos\psi}{2} - \frac{1+2\cos\psi}{1+\cos\psi}\ln(\eta + x_2 r_1 + x_1 r_2\cos\psi) + \frac{1}{1+\cos\psi}\ln(\eta + x_1 r_2 + x_2 r_1\cos\psi) ,$$

$$H_4 = H_2 + \frac{2\cos\psi}{1+\cos\psi}\ln[(1+\cos\psi)x_2 r_1] + \frac{1}{1+\cos\psi}\ln\frac{1+\cos\psi}{2} + \frac{1}{1+\cos\psi}\ln(\eta + x_2 r_1 + x_1 r_2\cos\psi) - \frac{1+2\cos\psi}{1+\cos\psi}\ln(\eta + x_1 r_2 + x_2 r_1\cos\psi) ,$$

$$\chi = 1 - x_1 - x_2 , \quad r_{1,2} = 1 - x_{1,2} .$$

The function Ξ and expressions η and $\cos\psi$ are defined in (38). For a derivation and details we refer the reader to [21].

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